

## Lecture 7: Frequency & Phase Modulation (FM & PM)

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EE423: Communication Electronics.

## Angle Modulation (FM & PM)

$$c(t) = A \cos(\omega_c t + \theta_0)$$

$$\varphi_{FM \text{ or } PM}(t) = A \cos \theta(t)$$

$$\omega_i(t) \triangleq \frac{d\theta(t)}{dt}$$

$$\theta_i(t) \triangleq \theta(t) - \omega_c t$$

$\theta(t)$ : **generalized angle** of modulated signal.

$\omega_i(t)$ : **instantaneous frequency** of modulated signal.

$\theta_i(t)$ : **instantaneous phase** of modulated signal.

## Frequency Modulation (FM)

- The *instantaneous frequency* of the modulated signal changes in proportion to the message.

$$\varphi_{FM}(t) = A \cos \left( \omega_c t + k_f \int_{-\infty}^t m(t) dt \right)$$

$$\theta_{FM}(t) = \omega_c t + k_f \int_{-\infty}^t m(t) dt$$

$$\omega_{i_{FM}}(t) = \omega_c + k_f m(t)$$

$$\theta_{i_{FM}}(t) = k_f \int_{-\infty}^t m(t) dt$$



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## Phase Modulation (PM)

- The *instantaneous phase* of the modulated signal changes in proportion to the message.

$$\varphi_{PM}(t) = A \cos \left( \omega_c t + k_p m(t) \right)$$

$$\theta_{PM}(t) = \omega_c t + k_p m(t)$$

$$\theta_{i_{PM}}(t) = k_p m(t)$$

$$\omega_{i_{PM}}(t) = \omega_c + k_p \frac{dm(t)}{dt} = \omega_c + k_p m'(t)$$



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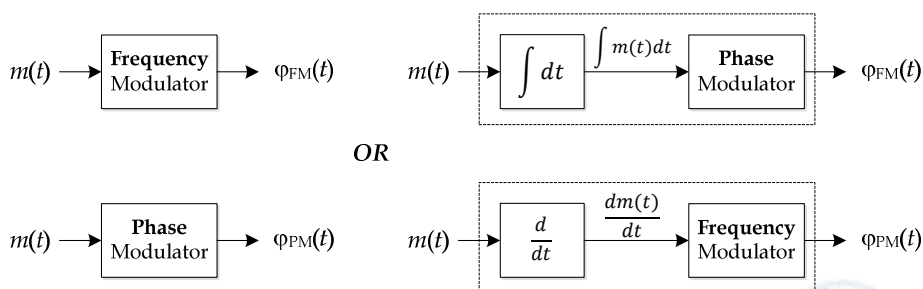
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## FM and PM Equivalence

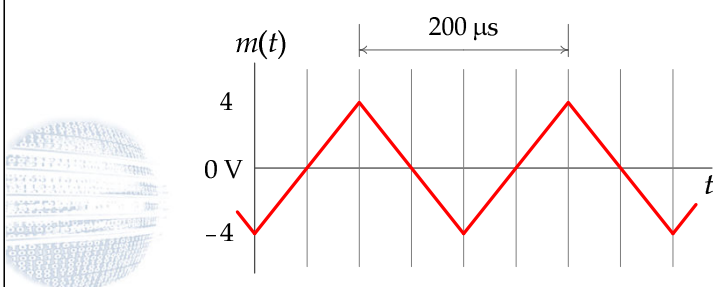
- FM
  - Constant amplitude  $A$
  - Constant carrier frequency  $\omega_c$
  - Variable *instantaneous* frequency  $\omega_i \propto m(t)$
  - Variable *instantaneous* phase  $\theta_i \propto \int m(t)dt$
- PM
  - Constant amplitude  $A$
  - Constant carrier frequency  $\omega_c$
  - Variable *instantaneous* frequency  $\omega_i \propto m'(t)$
  - Variable *instantaneous* phase  $\theta_i \propto m(t)$

## FM and PM Equivalence



## Example

- For the following message signal  $m(t)$  and a 100 MHz carrier:
  - Sketch the **FM** modulated signal. Use  $k_f = 2\pi \times 10^5$  rad/s/V.
  - Sketch the **PM** modulated signal. Use  $k_p = 5\pi$  rad/V.
  - Find  $\Delta f$  for both modulated signals.

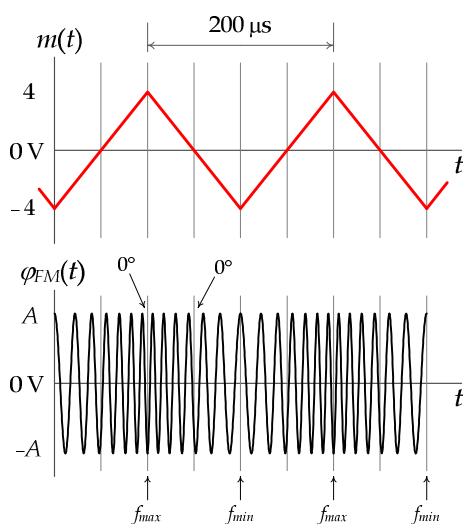


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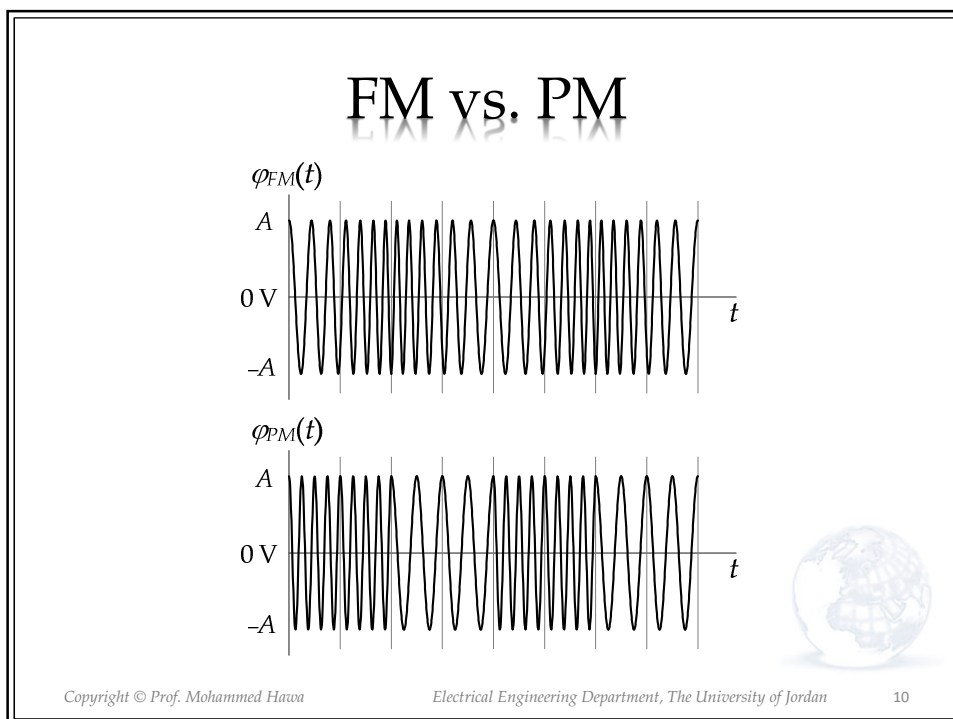
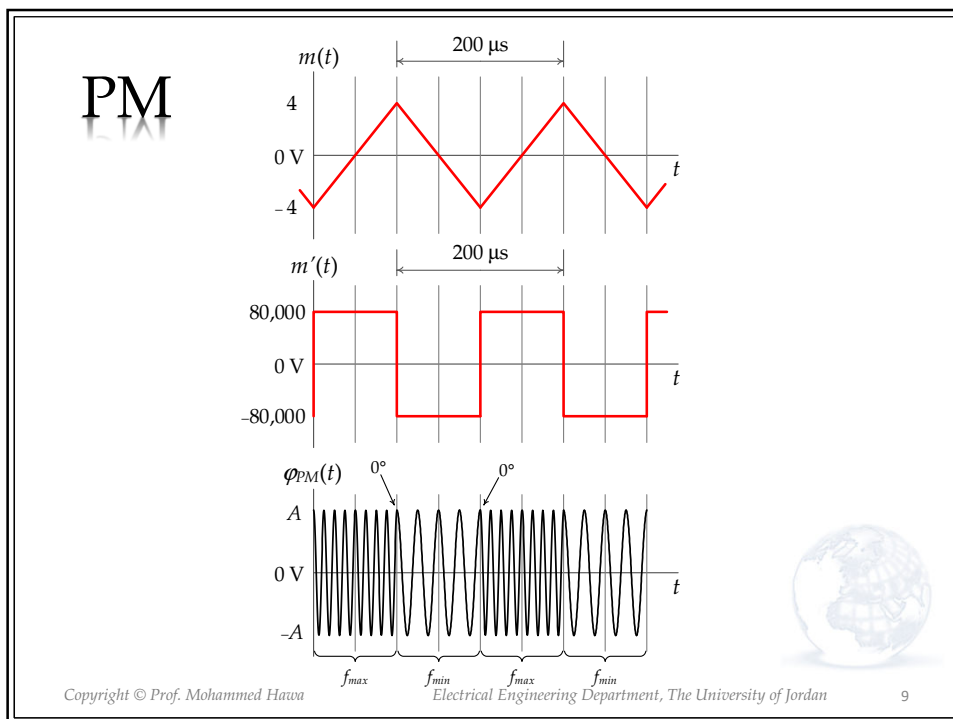
## Solution: FM



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## Peak Frequency Deviation

- For FM:

$$\Delta f \triangleq \frac{f_{max} - f_{min}}{2} = \frac{k_f}{2\pi} \times \frac{m(t)_{max} - m(t)_{min}}{2}$$

$$\Delta f = \frac{k_f}{4\pi} \times m(t)_{pk-pk} \quad [Hz]$$

- For PM:

$$\Delta f \triangleq \frac{f_{max} - f_{min}}{2} = \frac{k_p}{2\pi} \times \frac{m'(t)_{max} - m'(t)_{min}}{2}$$

$$\Delta f = \frac{k_p}{4\pi} \times m'(t)_{pk-pk} \quad [Hz]$$



## FM and PM Bandwidth

- Mathematically speaking:

- $B_{FM} = \infty$

- $B_{PM} = \infty$

- Practically speaking, use **Carson's Rule**:

- $B_{FM} \approx 2\Delta f + 2B = 2B(\beta+1)$

- $B_{PM} \approx 2\Delta f + 2B = 2B(\beta+1)$

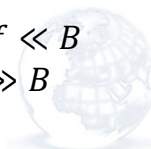
- FM Modulation Index:

- $\beta = \Delta f / B$

- *Narrow-Band FM* (NBFM) has  $\beta \ll 1$  or  $\Delta f \ll B$

- *Wide-Band FM* (WBFM) has  $\beta \gg 1$  or  $\Delta f \gg B$

- FM radio uses WBFM with  $\beta = 5$



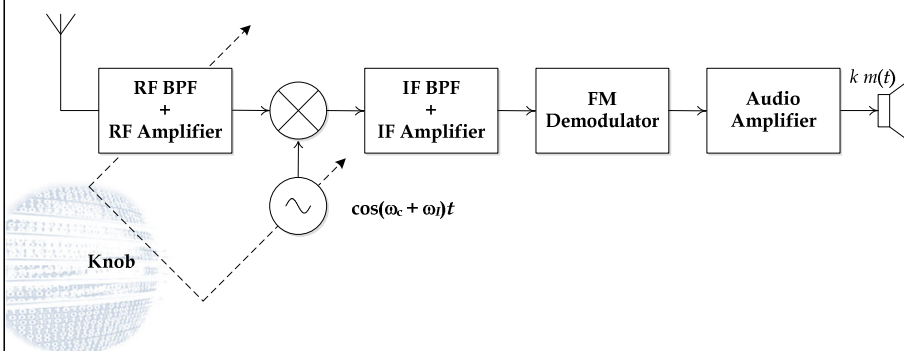
## Applications: FM Radio

- FM + FDM
  - The baseband message is 15 kHz (voice + music).
  - With  $\beta = 5$ , the bandwidth of each FM station is 200 kHz (both U.S. and Europe).
  - The broadcast range is 88 – 108 MHz.
- FM radio sounds better than AM radio:
  - $m(t)$  has a larger bandwidth.
  - WBFM: exchanging SNR for bandwidth.



## FM Superheterodyne Receiver

- IF frequency = 10.7 MHz
- L.O. frequency = 88 + 10.7 MHz to 108 + 10.7 MHz



## FM Hardware

- FM Modulator:
  - VCO (Voltage-Controlled Oscillator).
- FM Demodulator:
  - FM Discriminator (also called Slope Detector or Ratio Detector).
  - Quadrature Detector.
  - Phase-Locked Loop.

